G, $FC=2BC=4r\sin\frac{\pi}{n}$ is the radius and $\frac{2\pi}{n}$ the central angle, from G to H, $GD=3CD=6r\sin\frac{\pi}{n}$ is the radius and $\frac{2\pi}{n}$ the central angle and so on. For the nth side or the last side of the first revolution, radius $=2nr\sin\frac{\pi}{n}$, for the last side of second revolution radius $=4nr\sin\frac{\pi}{n}$, for the last side of the mth revolution radius $=2mnr\sin\frac{\pi}{n}$.

Hence without the use of calculus if $s=$ length of path, arc $AF=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$, arc $FG=4r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$, arc $GH=6r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$.

$\therefore\ s=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}\{1+2+3+\dots(mn-1)\}=2r\times\frac{2\pi}{n}\times\sin\frac{\pi}{n}\frac{mn(mn-1)}{2}$

$=2\pi rm(mn-1)\sin\frac{\pi}{n}$. But $r=1$, $m=120$.

$\therefore\ s=240\pi(120n-1)\sin\frac{\pi}{n}$.

If $n=6$, $s=271057.248$ feet, if $n=$ infinity $s=2(120)^2\pi^2=284245.9361$ feet.

Also solved by *ALFRED HUME* and the *PROPOSER*.

12. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Given the equations $2z^3=x+3z$ and $5z^2=y+2z$. To find $\frac{dy}{dx}$ for $x=0$.

Solution by Professor P. H. PHILBRICK, Lake Charles, Louisiana.

We have $x=2z^3-3z$, and $y=5z^2-2z$. $\therefore\ dx=(6z^2-3)dz$, $dy=(10z-2)dz$, and $\frac{dy}{dx}=\frac{10z-2}{6z^2-3}$.

But $x=2z^3-3z=z(2z^2-3)=0$. $\therefore\ z=0$, and $\pm\sqrt{\frac{3}{2}}$.

$\therefore\ \frac{dy}{dx}=\frac{2}{3}$, or $\pm\frac{5\sqrt{6}-2}{6}$, when $x=0$.

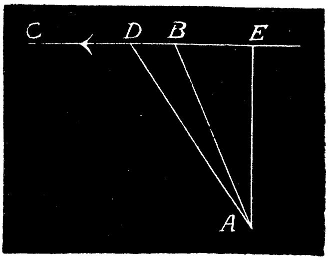
Also solved by Professors *BLACK, DRAUGHON, MATZ, SCHEFFER, WHITAKER*, and *ZERR*.

13. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

A steamer whose curse is due west and speed 10 knots is sighted by another steamer going at 8 knots; what course must the latter steer, so as to cross the track of the former at the least possible distance from her?

Solution by C. W. M. BLACK, A. M., Department of Mathematics, Wilmington Conference Academy, Dover, Delaware.

Let A be the position ot sighting steamer, D of other when sighted, B where A crosses course of D, which by that time has reached C. AE is $CE\perp$. Let $y=CB=CD+DB=CD+DE-BE$. Let $a=$no. hrs. A takes to reach B, and $b=AE$. $\angle EAB=x$, $\angle EAD=\theta$.



$AE'=AB\cos x$, or $b=8a\cos x$, $a=\dfrac{b}{8\cos x}$. $CD=10a$.

$DE=b\tan\theta$, $BE=b\tan x$. $y=\dfrac{10b}{8\cos x}+b\tan\theta-b\tan x$. $\dfrac{dy}{dx}=\dfrac{5b\sin x}{4\cos^2 x}-b\sec^2 x$.

Let $\dfrac{dy}{dx}=0$, whence $\sin\theta=\dfrac{4}{5}$, $\theta=53^\circ\,8'$.

The equation shows that b and θ do not affect the result.

Also solved by *Professors PHILBRICK, WHITAKER, ZERR*, and the *PROPOSER*.

14. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Right triangles are inscribed in a circle whose center $=(a,b)$, and radius$=c$. If one of the legs passes through a fixed point, prove that $c^2(x^2+y^2)=(a^2+b^2-c^2-ax-by)^2$ is the curve to which the other leg is always tangent; the fixed point being the origin of the co-ordinates.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $lx+my=1\ldots.(1)$ be the chord of the circle whose envelope is to be found. Then $y=\dfrac{m}{l}x\ldots.(2)$ is the side of the triangle passing through the fixed point, and $(x-a)^2+(y-b)^2=c^2\ldots.(3)$ is the equation to the circle.

(1) and (2) intersect in $x'=\dfrac{l}{l^2+m^2}$, $y'=\dfrac{m}{l^2+m^2}$.

This point being on (3), we must have on substituting x' and y' in (3), after reducing, $(a^2+b^2-c^2)(l^2+m^2)-2al-2bm+1=0\ldots.(4)$. Making (4) homogeneous in l and m by means of (1) and arranging,

$$[x^2-2ax+(a^2+b^2-c^2)]\frac{l^2}{m^2}+2(xy-bx-ay)\frac{l}{m}+[y^2-2by+(a^2+b^2-c^2)]=0\ldots.(5),$$

a quadratic in the parameter $\dfrac{l}{m}$, giving the envelope

$$c^2(x^2+y^2)=[(a^2+b^2-c^2)-by-ax]^2\ldots.(6).$$

Also solved by *P. S. BERG, G. B. M. ZERR* and the *PROPOSER*.

15. Proposed by CHARLES E. MYERS, Canton, Ohio.

From a given quantity of material a cylindrical cup with circular bottom and open top is to be made, the cup to contain the greatest amount. What must be its dimensions?

Solution by F. P. MATZ, M. S., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent the radius of the base by x, and the altitude by y; then, obviously, $\pi x^2+2\pi xy=s$. $\therefore y=(s-\pi x^2)\div 2\pi x$.

Also, $V=\pi x^2 y=\pi x^2\left(\dfrac{s-\pi x^2}{2\pi x}\right)$, $=$a maximum.